UC Davis Biometeorology Group

Soil Heat Flow and Temperature

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Question: Why do we want to know the soil heat flow and temperature?

Answer: We need _G_ at the surface to determining the energy balance

Neglecting small energy components, the energy balance on a surface can be expressed using the following equation, where R_n is net radiation, G is the soil heat flux density, H is the sensible heat flux density, and LE is the latent heat flux density.

NET RADIATION   =   SOIL HFD   + SENSIBLE HFD   +   LATENT HFD

$$R_N \quad = \quad G \quad + \quad H \quad + \quad LE$$

In this document, the methodology to estimate G at the surface is discussed.

DEFINITIONS

1)   Temperature (T) is a measure of the heat stored (°C or K)

2)   Upper temperature (T_1) is the temperature at depth z_1

3)   Lower temperature (T_2) is the temperature at depth z_2

4) Volumetric Heat Capacity is the amount of heat required to raise the temperature of a unit volume by one Kelvin ($J \ m^{-3} \ K^{-1}$)

5) Thermal Conductivity (C_1) is the ratio of the heat flux density to the temperature gradient in ($W \ m^{-1} \ K^{-1}$)

SOIL HEAT FLUX DENSITY

1) Soil heat flux density (G) is the conduction of energy per unit area in response to a temperature gradient. For small depth changes,

$$G \approx -C_1 \left(\frac{\partial T}{\partial z} \right)$$

(1)

Here, the thermal conductivity $C_1 = KA$, where K is the heat conductivity of the material in $W \ m^{-3} \ K^{-1}$ and A is the surface area in m^2, so for heat flow through a unit surface area C_1 has the units $W \ m^{-1} K^{-1}$. In Eq. 1, is positive when the temperature decreases with depth in the soil. The negative sign is included to make G positive when heat is transferring downward. Because of instrument limitations, it is not possible to accurately measure the temperature gradient unless there is sufficient distance between the sensors. Consequently, G is estimated as

$$G \approx -C_1 \left(\frac{T_2 - T_1}{z_2 - z_1} \right)$$

(2)

where z_2 is sufficiently far below z_1 to allow for a measurable difference between T_2 and T_1. Equation 2 assumes that C_1 is constant with depth in the soil.

2) Thermal Diffusivity (k) is the ratio of the thermal conductivity to the volumetric heat capacity.

$$\kappa = \frac{C_1}{\rho_s C_p} = \frac{C_1}{C_V}$$

(3)

C_1=the thermal conductivity ($W \ m^{-1} \ K^{-1}$)

r_s = the apparent soil density ($kg \ m^{-3}$)

C_p = the apparent mass specific heat capacity (J kg^{-1} K^{-1})

C_V = volumetric heat capacity (J m^{-3} K^{-1})

k = thermal diffusivity (m^2 s^{-1})

where $C_V = \rho_s C_p$

$$(4)$$

So the thermal conductivity in terms of diffusivity and volumetric heat capacity is

$$C_1 = \kappa \rho_s C_p = \kappa C_V$$

$$(5)$$

Therefore, G can be expressed in terms of diffusivity, volumetric heat capacity, and the temperature gradient as: $G = -\kappa C_V \dfrac{\partial T}{\partial z}$

$$(6)$$

In terms of the temperature gradient, G is

$$G = -\kappa C_V \left(\frac{T_2 - T_1}{z_2 - z_1} \right)$$

$$(7)$$

The variable k is useful as a measure of how fast the temperature of a soil layer changes. The rate at which the heat content of a layer of soil changes depends on the volumetric heat capacity (C_V) and the rate of temperature change of the soil volume per unit time. For a unit surface area, the rate of change in heat storage within the soil layer is expressed as $-C_V \dfrac{\partial T}{\partial t} \Delta z$. For a unit surface area, the rate of change in heat storage within the soil is also equal to the change in heat flux density through the soil layer $\dfrac{\partial G}{\partial z} = \dfrac{\partial}{\partial z}\left(-C_1 \dfrac{\partial T}{\partial z} \right)\Delta z$. Assuming that the physical properties of the soil are constant with depth in the soil and equating these two expressions, we get $-C_V \dfrac{\partial T}{\partial t}\Delta z = -C_1 \dfrac{\partial}{\partial z}\left(\dfrac{\partial T}{\partial z} \right)\Delta z$ which simplifies to $\dfrac{\partial T}{\partial t} = \dfrac{C_1}{C_V} \dfrac{\partial}{\partial z}\left(\dfrac{\partial T}{\partial z} \right) = \kappa \dfrac{\partial^2 T}{\partial z^2}$

$$(8)$$

Therefore, k is useful to determine the rate of temperature change of a soil layer. Recall that k is directly proportional to C_1 and inversely proportional to C_v. Both C_1 and C_v increase as the water content of the soil increases; however, C_1 increases more rapidly with water content when the soil is dry and it slows as the soil becomes wet. C_v continues to increase even when the soil is relatively wet. Consequently, for a dry soil, k increases with water content, but it slows and sometimes decreases when the soil nears saturation. As a result, the maximum change in temperature with time will occur at a water content below saturation.

Question: How do you determine $G = G_1$ at the surface?

The change in heat storage of a layer of soil between the surface $z_1=0$ and some depth z_2 is given by

$$(9)$$

where G is positive downward into the soil.
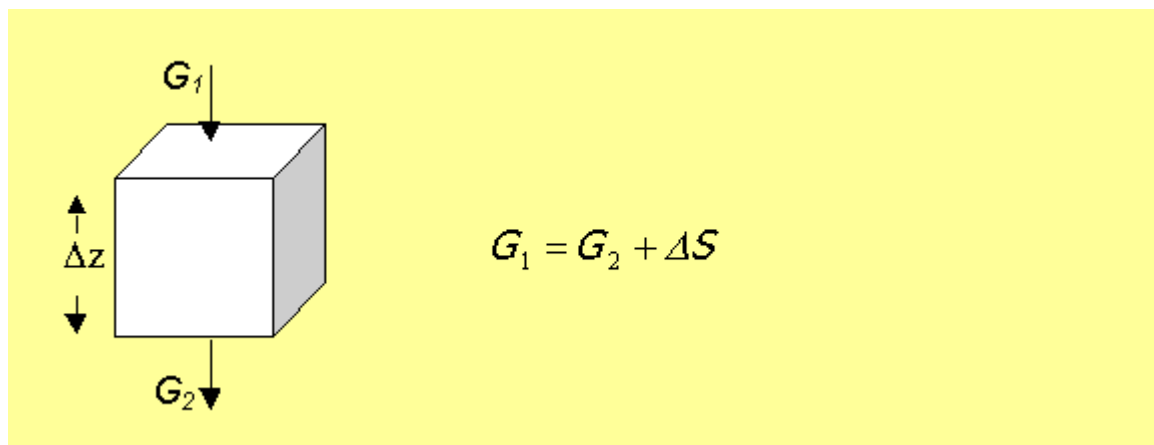
Note: Soil heat flux density is positive downwards. When $DS = -(G_2 - G_1)$ is positive, then more heat is entering the top than leaving the bottom of the soil layer and the soil is warming. If $DS = -(G_2 - G_1)$ is negative, then more heat is leaving than entering the layer and the soil is cooling.

Examples:
Let $G_1 = 100$ W m^{-2} and $G_2 = 50$ W m^{-2}, then
$\Delta S = -(50-100) = 50$ W m^{-2} and the soil is warming

Let $G_1 = -100$ W m^{-2} and $G_2 = -50$ W m^{-2}, then
$\Delta S = -[-50-(-100)] = -50$ W m^{-2} and the soil is cooling

Let $G_1 = -50$ W m^{-2} and $G_2 = 10$ W m^{-2}, then
$\Delta S = -[10-(-50)] = -60$ W m^{-2} and the soil is cooling

Let $G_1 = 50$ W m^{-2} and $G_2 = -10$ W m^{-2}, then
$\Delta S = -[-10-50] = 60$ W m^{-2} and the soil is warming

Accounting for the change in soil heat storage is important for estimating G at the surface. This is clearly indicated in Fig. 1, which shows a plot of G_2 + DS versus G_2 for half hour measurements of G_2 at 0.02 m depth under a cover of about 0.1 m tall, cool-season grass. The regression equation indicates that the soil heat flux density at the surface is under-estimated by about 40% when not corrected for the change in soil heat storage.
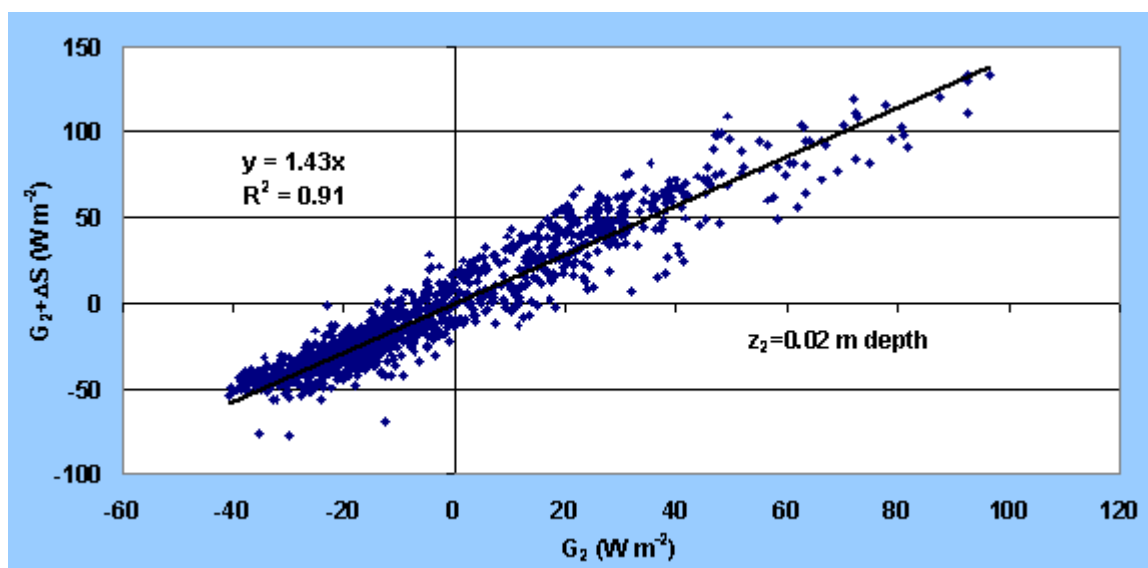

FIGURE 1. SURFACE SOIL HEAT FLUX DENSITY (G) CALCULATED USING A HEAT FLUX PLATE AT 0.02 M DEPTH (G_2) AND THE CHANGE IN HEAT STORAGE (DS) ESTIMATED FROM TEMPERATURE CHANGE AT 0.01 M DEPTH VERSUS G_2 ALONE USING HALF HOUR DATA COLLECTED UNDER 0.10 M TALL, COOL-SEASON GRASS. THE SOIL WAS A YOLO CLAY LOAM SOIL THAT WAS IRRIGATED ONCE PER WEEK.

CHANGE IN HEAT STORAGE

The net change of heat storage per unit time (DQ/Dt) in **J s^{-1}** within a soil volume (V) is given by:

$$\frac{\Delta Q}{\Delta t} = C_V \frac{\partial T}{\partial t} \cdot V \approx C_V \frac{T_f - T_i}{t_f - t_i} V$$

(10)

J s^{-1} = (J m^{-3} K^{-1}) (K s^{-1}) (m^3)

where T_f is the final temperature at time t_f and T_i is the initial temperature at time t_i.

Because the change in stored soil heat (DS) per unit surface area is equal to the net rate of heat storage divided by the soil surface area and $V/A=Dz$, we can calculate DS in **W m^{-2}** as

$$\Delta S = \frac{\Delta Q}{\Delta t} \frac{1}{A} = C_V \left(\frac{T_f - T_i}{t_f - t_i} \right) \frac{V}{A} = C_V \left(\frac{T_f - T_i}{t_f - t_i} \right) \Delta z$$

(11)

If G_2 is measured and DS is calculated using Eq.11, then G (= G_1 at the surface) is calculated using:

$$G = G_1 = G_2 + \Delta S$$

(12)

Example: Given a uniform soil with $C_V = r_s C_p = 1.677 \times 10^6$ J m^{-3} K^{-1}, a soil heat flux plate placed at an 0.08 m depth, and soil-averaging temperature sensors located at 0.02 and 0.06 m depth to represent the change in heat storage between the surface and 0.08 m depth. If the heat flux plate measurement averages G_2 = 20 W m^{-2} and the average soil temperature from the two sensors increases by 1 K during a one-hour period, what is the soil heat flux density ($G = G_1$) at the surface?

$$\Delta S = C_V \left(\frac{T_f - T_i}{t_f - t_i} \right) \Delta z$$

$$\Delta S = \left(1.677 \times 10^6 \, Jm^{-3}K^{-1} \right) \left(\frac{1K}{3600s} \right) (0.08m) = 37.3 Wm^2$$

$$G = G_1 = G_2 + \Delta S = 20 + 37.3 = 57.3 \text{ W m}^{-2}$$

Question: Using the same criteria as above but now with the mean soil temperature dropping by 1 K, what is the soil heat flux density ($G = G_1$) at the surface?

$$\Delta S = C_v \left(\frac{T_f - T_i}{t_f - t_i} \right) \Delta z$$

$$\Delta S = \left(1.677 \times 10^6 \, Jm^{-3}K^{-1}\right)\left(\frac{-1K}{3600s}\right)(0.08m) = -37.3 Wm^2$$

$$G = G_1 = G_2 - \Delta S = 20 + (-37.3) = -17.3 \text{ W m}^{-2}$$

If the soil heat flux density at 0.08 m depth is G = 20 W m^{-2} and the temperatures 20°C and 20.8°C are measured on either side of the heat flux plate. What is the thermal conductivity (C_1) if the temperature sensors are 0.02 m below and above the heat flux plate? Which temperature is recorded above the heat flux plate?

T=20.8°C is the upper temperature because G>0

  W m^{-2}K^{-1}

Thermal Properties of Soils

Heat capacity depends on the mineral, organic matter, and water content of a soil. The apparent heat capacity (C_v) on a volume basis has the units J m^{-3} K^{-1} and on a mass basis (C_p) it has the units J kg^{-1} K^{-1}. Equation 13 is used to express heat capacity.

$$C_v = \rho_s C_p = c_{pav}\rho_b + c_{pw}\rho_b \theta_m$$

(13)

J m^{-3}K^{-1} = (kg m^{-3})(J kg^{-1}K^{-1})

where r_s is the apparent density in kg of moist soil per m^3, r_b is the bulk density of the soil in kg of dry soil per m^3, c_{pav} is the average heat capacity on a mass basis for the solid constituents of the soil in J per kg of dry soil per Kelvin, q_m is the water content on a mass basis in kg of water per kg of dry soil, and c_{pw} is the heat capacity of water on a mass basis in J per kg of H$_2$0 per Kelvin.

The heat capacity of the solid constituents of the soil (C_{pav}) depends on the amount of sand, clay, silt, and organic matter in the soil. For most mineral soils,

$$c_{pav} \approx 837$$

$$\text{J kg}^{-1}\text{K}^{-1}$$

The heat capacity of the water component of the soil (q_{mcpw}) depends on the water content and the heat capacity for water.

$$c_{pw}\rho_b\theta_m = \frac{\rho_w}{\rho_w}\left(c_{pw}\rho_b\theta_m\right) = \left(\rho_w c_{pw}\right)\rho_b\left(\frac{\theta_m}{\rho_w}\right)$$

$$\text{J kg}^{-1}\text{K}^{-1}$$

The volumetric heat capacity is $r_w c_{pw}$, which is approximated as

$$\text{J m}^{-3}\text{K}^{-1}$$

and the volumetric water content q_v in m³ H₂O per m³ of dry soil is

$$\theta_v = \rho_b\left(\frac{\theta_m}{\rho_w}\right) \Rightarrow \frac{kg\,dry\,soil}{m^3}\cdot\frac{kg\,H_2O}{kg\,dry\,soil}\cdot\frac{m^3\,H_2O}{kg\,H_2O} \Rightarrow \frac{m^3\,H_2O}{m^3}$$

Therefore, the heat capacity of the soil component is approximated as

$$c_{pav}\rho_b \approx 837\,\rho_b$$

and the heat capacity of the water component is approximated as

$$\left(\rho_w c_{pw}\right)\rho_b\left(\frac{\theta_m}{\rho_w}\right) = \left(\rho_w c_{pw}\right)\theta_v \approx 4.19\times 10^6\,\theta_v$$

Substitution into Eq. 13 gives the following approximation for C_v.

$$C_v = 837\,\rho_b + 4.19\times 10^6\,\theta_v$$

$$\text{J m}^{-3}\text{K}^{-1} \qquad (15)$$

Because we are interested in heating and cooling the soil, it is useful to have an expression for the amount of energy (Q) needed to raise or lower the temperature of a known volume (V) of soil from T_i to T_f. The equation is

$$Q = C_v\left(T_f - T_i\right)V$$

$$J = J\ m^{-3}\ K^{-1}\ (K)\ m^3 \qquad (16)$$

Problems:

1) Given a wet soil at T_i = 18°C with q_v = 0.23 and r_b = 1300 kg m^{-3}. Find the amount of heat required to raise a 1 m^3 volume of soil to a temperature T_f = 20°C for a surface area of 1 m^2 to a depth of 1 m.

$$Q = (837\ \rho_b + 4.19 \times 10^6\ \theta_v)\ (T_i - T_f)\ V$$

$$Q = (837 \times 1300 + 4.19 \times 10^6 \times 0.23)\ (20 - 18)\ (1.0\ m^3)$$

$$Q = (1.088100 \times 10^6 + 0.963700 \times 10^6)\ (2)\ (1) = 4.10 \times 10^6\ J = 4.10\ MJ$$

2) Given a wet soil with heat flux G = 25 W m^{-2} into the soil surface (q_v = 0.18, r_b = 1300 kg m^{-3} and $Q = G\ A\ t$). Find the average temperature increase that occurs in one day in the top 1 m of soil assuming that the heat is evenly distributed in the 1 m soil layer.

$$Q = C_v\ V\ \Delta T$$

$$Q = G\ A\ t$$

$$\Delta T = \frac{G \cdot A \cdot t}{C_v \cdot V}$$

$$\Delta T = \frac{25\ J\ s^{-1} m^{-2} \times 1 m^2 \times 86400\ s\ d^{-1} \times 1 d}{(837 \times 1300 + 4.19 \times 10^6 \times 0.18) \times 1} = \frac{2.16 \times 10^6}{1.84 \times 10^6} = 1.17\ °C$$

3) Given a wet soil with q_v = 0.18, r_b = 1300 kg m^{-3}, $Q = G\ A\ t$, and the average daily net radiation is R_n = 150 W m^{-2}, find the ratio G/R_n if the increase in average soil temperature to 1 m depth over a month's time is 10°C.

$$G = \frac{Q}{A \cdot t} = \frac{C_v V (T_f - T_i)}{A \cdot t}$$

From problem 2, we know that $Cv \cdot V$ = 1.8410^6 J m^{-3} K^{-1}

$$G = \frac{1.84 \times 10^6 \ J m^{-3} K^{-1} \times 1 \ m^3 \times 10K}{1 m^2 \times 1 mo \times 86400s \ d^{-1} \times 30d \ mo^{-1}} = 7.1 \ W m^{-2}$$

$$\frac{G}{R_n} = \frac{7.1}{150} = 0.047 = 4.7\%$$

4) Given a soil with the following temperatures and properties, find the quantity of heat stored in the soil per unit area (Q/A), for the period from 0500 to 1400 hrs.

Soil Depth	T_i	T_f	r_b	q_v
m	0500 h	1400 h	kg m^{-3}	
0.00-0.05	15	35	1.0	0.05
0.05-0.20	20	30	1.1	0.10
0.20-0.60	18	25	1.2	0.20
0.60-1.20	17	18	1.3	0.25

$Q/A = [(837 \quad \times \quad \rho_b \quad + \quad 4.19 \times 10^6 \times \theta_v) \times V \times \Delta T] / A$
$\quad$ (J kg^{-1}K^{-1}) (kg m^{-3}) (J m^{-3}K^{-1}) $\quad$ (m^3) (K) (m^2)

0.00-0.05 m
$Q/A=[(837 \times 1000 + 4.19 \times 10^6 \times 0.05) \times 0.05 \times 20]/1 = 1.048 \times 10^6 = 1.05$ MJ m^{-2}
0.05-0.20 m
$Q/A=[(837 \times 1100 + 4.19 \times 10^6 \times 0.10) \times 0.15 \times 10]/1 = 2.013 \times 10^6 = 2.01$ MJ m^{-2}
0.20-0.60 m
$Q/A=[(837 \times 1200 + 4.19 \times 10^6 \times 0.20) \times 0.40 \times 7]/1 = 5.165 \times 10^6 = 5.17$ MJ m^{-2}
0.60-1.20 m
$Q/A=[(837 \times 1300 + 4.19 \times 10^6 \times 0.25) \times 0.60 \times 1]/1 = 1.283 \times 10^6 = 1.28$ MJ m^{-2}

Total heat stored = 1.05 + 2.01 + 5.17 + 1.28 = 9.51 MJ m^{-2}