UC Davis Biometeorology Group

Sensible Heat Flux

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Air molecules move about at near the speed of sound (300 m s^{-1}), and they are constantly colliding with their surroundings and each other. In general, if a molecule travels from some height z to height $z + l$, where it collides with another molecule, the distance l is called the "mean free path". After colliding with the new molecule, the energy (E_k) is transferred to the new molecule and the temperature increases to that of the original air molecule.

$$E_k = -m C_p \left[T(z+l) - T(z)\right] \qquad \text{Joules}$$

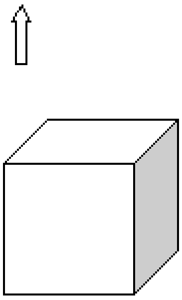
where m is the mass of the molecule (kg) and C_p (J kg^{-1}K^{-1}) is the specific heat per unit mass at constant pressure. Using a Taylor's approximation, we can write

$$T(z+l) = T(z) + \frac{\partial T}{\partial z} l \qquad \text{Kelvin}$$

and, by substitution, we get

$$E_k = -m C_p \frac{\partial T}{\partial z} l \qquad \text{Joules}$$

To know the heat transfer per unit area, we need to know the number of active molecules moving with a vertical velocity component at any given time. If we let $\frac{N}{V}$ equal the number of molecules per unit volume, then, at any given time, $\frac{N/3}{V}$ is the number with a vertical velocity. The flux (F) of molecules crossing a horizontal plane per unit time is the product of $\frac{N/3}{V}$ and the average vertical velocity (c) of the active molecules.



$$\frac{N/3}{V} = \frac{\#active\ molecules}{unit\ volume}$$

$$F = \frac{cN/3}{V}$$

The heat flux density equals the product of the molecular flux density and the heat transferred by each molecule.

$$H = -F\left(mC_p\frac{\partial T}{\partial z}l\right) = -\left(\frac{cN/3}{V}\right)\left(mC_p\frac{\partial T}{\partial z}l\right) \quad \text{W m}^{-2}$$

However, the product of the number of active molecules and the mass per molecule (Nm) is the mass of air per unit volume or density ($\rho = \frac{Nm}{V}$), so by rearranging terms, we get a formula for sensible heat flux density (H)

$$H = \left(\frac{-cl}{3}\right)\left(\frac{Nm}{V}\right)C_p\frac{\partial T}{\partial z} = \frac{-cl}{3}\rho C_p\frac{\partial T}{\partial z} = -\kappa\rho C_p\frac{\partial T}{\partial z} \quad \text{W m}^{-2}$$

Here, $\kappa = cl/3$ is the “thermal diffusivity” in m^2 s^{-1}. A typical value for air near Earth’s surface is $k = 2.2 \times 10^{-3}$ m^2 s^{-1}.

Because of sensor limitations, it is not possible to measure temperature differences over small distances. Consequently, we measure the temperature at two heights and assume that the gradient is constant between the two levels. Then we use the following equation

$$H = -\kappa\rho C_p\left(\frac{T_2 - T_1}{z_2 - z_1}\right) \quad \text{W m}^{-2}$$

where z_2 is further from the surface than z_1. Also, we usually don’t know the value for k, so we use the substitution

$$\frac{1}{r_h} = \frac{\kappa}{z_2 - z_1} = g_h \qquad \text{m s}^{-1}$$

to get *H* in terms of a conductance (g_h) or resistance (r_h) to sensible heat transfer.

$$H = -\rho C_p \left(\frac{T_2 - T_1}{r_h} \right) \qquad \text{W m}^{-2}$$

The conductance (g_h) is the rate at which one cubic meter of heat passes through one square meter of surface area. Conductance has the units m s^{-1} and resistance has units s m^{-1}.

Temperature gradients are often large near a surface but become smaller with distance from the surface. Therefore, the temperature approaches a fixed value (T_∞) if the distance is far relative to the size of the object. The zone between the surface and where the temperature reaches (T_∞) is called the boundary layer for sensible heat flux. One could estimate *H* using (T_∞) and the surface temperature (*T*) as

$$H = -\kappa \rho C_p \left(\frac{T_\infty - T}{\delta} \right) \qquad \text{W m}^{-2}$$

where *d* is the "boundary layer height" or distance from the surface to where the temperature is near . However, *d* is difficult to measure, so we assume that *d* is related to the ratio of the object dimension *d* to the boundary layer height (*d*). We then define a dimensionless "Nusselt" number as

$$Nu = \frac{d}{\delta}$$

Because 1/*d* = *Nu*/*d*, we get H as

$$H = -\rho C_p \kappa Nu \frac{T_2 - T_1}{d} \qquad \text{W m}^{-2}$$

where *d* is the object dimension. The meaning of *d* depends on the shape of the object. For a flat plate, *d* is length parallel to the flow. For cylinders that are normal to the flow, *d* is the diameter. For spheres, *d* is the diameter. Values for *Nu* are determined experimentally.

When air blows around an object (forced convection), the flow can be turbulent or viscous (laminar). The wind speed and size and shape of the object determine if the flow is viscous or turbulent. At low wind speed, the flow is generally laminar, but it becomes turbulent at higher velocities. The dimensionless “Reynolds” number (*Re*) is related to onset of turbulence.

$$Re = \frac{u \cdot d}{\nu}$$

where *n* is the kinematic viscosity (»1.5 ´10^{-5} m^{2} s^{-1} for air), *d* is the object dimension in m, and *U* is the wind speed (m s^{-1}) relative to the object. R_e represents the ratio of inertial forces (related to acceleration of the fluid) to viscous forces (due to molecular transport of momentum). At low wind speeds, the inertial and viscous forces are similar, *Re* is small and the flow is orderly. For high *Re*, inertial forces are much larger than viscous forces and the airflow becomes turbulent.

The distance from an object where the temperature approaches T_∞ depends on whether the airflow is viscous or turbulent. Therefore, *Nu* is a function of *Re*. When air is blowing around an object, *Nu* can be estimated from *Re* as

$$Nu = b\, Re^{n} + c$$

where *b*, *c*, and *n* are empirically determined for a given object shape.

From Monteith (1973)

Object	Re	Nu
Flat Plate w/parallel flow	< 2 ´10^{4}	0.60 Re$^{0.5}$
	> 2 ´10^{4}	0.03 Re$^{0.8}$
Cylinders w/normal flow	10^{-1} – 10^{4}	0.32 + 0.51 Re$^{0.52}$
	10^{3} – 5 ´10^{4}	0.24 Re$^{0.60}$
	4 ´10^{4} – 4 ´10^{5}	0.024 Re$^{0.81}$
Spheres	0 – 300	2 + 0.54 Re$^{0.5}$
	50 – 1.5 ´10^{5}	0.34 Re$^{0.6}$

When the air around an object is not moving, the object may heat or cool rapidly. This causes a density change near the surface, which results in free or natural convection of heat. Since there is no wind, there is no *Re*. Then *Nu* becomes a function of the “Grashof” number (*Gr*).

$$Gr = \frac{a_t g d^3 |T - T_\infty|}{\nu^2}$$

where g is the acceleration due to gravity (9.8 m s^{-2}) and a_t is the coefficient of expansion (a_t=1/273 for an ideal gas). The Grashof number represents the product of the ratio of buoyancy to viscous forces and the ratio of inertial to viscous forces. At higher temperatures, *Gr* is bigger and there is more free convection. For air, *Gr* is approximately

$$Gr = \left|\left(1.58\times10^8\right)d^3\left(T - T_\infty\right)\right|$$

Nu is determined from *Gr* as

$$Nu = bGr^m$$

where b and m are determine empirically. The object dimension d is the smallest of length or width for horizontal flat plates, length for vertical flat plates, and diameter for cylinders and spheres.

Object	Gr	Nu
Horizontal Flat Plate	$< 10^5$	$0.60\ Gr^{0.25}$
	$> 10^5$	$0.13\ Gr^{0.33}$
Horizontal Cylinder	$10^4 – 10^9$	$0.48\ Gr^{0.25}$
	$> 10^9$	$0.09\ Gr^{0.3}$
Vertical Flat Plate	$10^4 – 10^9$	$0.58\ Gr^{0.25}$
	$10^9 – 10^{12}$	$0.11\ Gr^{0.3}$
Spheres	$< 2\ 10^{10}$	$2 + 0.54\ Gr^{0.25}$