Radiation

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Electromagnetic radiation is one method for transfer of energy without the need for a medium. The radiation emitted can be modeled as a function of frequency or wavelength using Planck's function, where h and k are constants and c is the speed of light. Recall that:

Speed of light = wavelength x frequency **(*c* = *ln*)**

$$E_\lambda(T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT} - 1\right)}$$

h=6.626 x 10^{-34} J s Planck's constant

k = 1.3806 x 10^{-23} J K^{-1} Boltzmann's constant

c = 3.0 x 10^{8} m s^{-1} Speed of light

The energy emitted by a black body source is a function of the 4^{th} power of the temperature in Kelvin following the Stefan-Boltzmann law.

$$E = \sigma T^4$$

The maximum wavelength (l_{max}) for energy emitted depends on the temperature as defined by Wein's law, where the wavelength is in mm and the temperature is in Kelvin.

$$\lambda_{max} = \frac{2890}{T}$$

Solar $\lambda_{max} = \frac{2890}{6000} = 0.48$ mm

Earth $\lambda_{max} = \frac{2890}{288} \approx 10.0$ mm

The Stefan-Boltzmann law applies to 'black' bodies, which emit the maximum possible energy at all wavelengths. For 'gray' bodies, which are not perfect emitters, an emissivity factor (0<e£ 1.0) is included.

$$E = \varepsilon \sigma T^4$$

For a black body, *e* = 1.0. For a gray body *e* < 1.0.

According to Kirchoff's Law, a body that is a good emitter at any given wavelength is also a good absorber at that wavelength.

Emissivity = *Absorptivity*

RADIATION SYMBOLS AND TERMS

Direct Solar Radiation (Q_s) – short wave band radiation energy flux density (W m^{-2}) received at the surface

Diffuse Solar Radiation (q) – diffuse or scattered short wave band radiation energy flux density (W m^{-2}) received at the surface

Upward Long Wave Radiation (L_u) – long wave radiation (W m^{-2}) upward from the surface.

Downward Long Wave Radiation (L_d) – long wave radiation (W m^{-2}) downward from the sky

Net Radiation (R_n) – the net amount of total radiation (W m^{-2}) absorbed by the surface

$R_n = (1 - a)(Q_s + q) + L_u + L_d$

DIRECT BEAM RADIATION

Outside of the Earth's atmosphere, the flux density of solar radiation is called the 'solar constant' (Q_c = 1367 W m^{-2})

As the radiation passes through the atmosphere, some of the radiation is scattered and the flux density that is transmitted can be estimated using an extinction equation

$$Q = Q_c \exp(-k_a x)$$

where x is the path length of the direct radiation through the atmosphere and k_a is the extinction coefficient, which depends on turbidity of the atmosphere. The flux density can also be determined using a transmission coefficient (t_a), where $0.6 < t_a < 0.9$ is the typical range.

$$Q = Q_c \tau_a^{\sec(a)}$$

For our purposes we will use the transmission equation to avoid the need to determine path length through the atmosphere. Note that Q is the flux density flow of energy from the sun ($J\ s^{-1}$) per unit area perpendicular to the sun's rays. If the energy is received by a surface that is not perpendicular to the sun's rays, a correction for the angle of incidence is required.

To determine the flux density received by an object, the flow (or flux) of direct beam short wave radiation from the sun is divided by the projection area (A_p = the projected area of an object that is perpendicular to the sun's rays). On the other hand, the flow or flux of the direct radiation (R_{ab}) is calculated as:

$$R_{ab} = Q A_p$$

If we know R_{ab}, then the amount of energy received per unit surface area of the object is calculated using the "interception factor" (A_p/A) as

$$Q_s = \frac{R_{ab}}{A} = \left(\frac{A_p}{A}\right)Q$$

The surface area of an object is estimated using geometry, but determining A_p is often difficult. However, it is relatively simple to determine A_p from the area of the object shadow on a horizontal surface (A_h). After measuring the area of the object shadow, then A_p is calculated as

$$A_p = A_h \cos(a)$$

where a is the zenith angle. This relationship comes from trigonometry where the $\cos(a)$ accounts for the fact that the length of one dimension of the horizontal shadow area is increased by the factor $\cos(a)$.

The zenith angle (a) is calculated using the formula

$$\cos(\alpha) = \sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(\omega t)$$

where f is the site latitude in radians, d is the angle of the sun relative to the equatorial plane, w is the angular velocity of Earth's rotation (15^{o} h^{-1} = 0.2618 rad h^{-1}) and t is the time in hours relative to solar noon.

$t = h - 12$

where h is military time (h = 0-24 starting at midnight). Using the above relationships, we can determine the flux density of direct radiation received per unit surface area as

$$Q_s = \frac{R_{ab}}{A} = \left(\frac{A_p}{A}\right)Q = \left(\frac{A_p \cos(\alpha)}{A}\right)Q$$

DIFFUSE RADIATION

Diffuse radiation is energy that is scattered by the atmosphere. Some of this radiation is received by the surface, but it is not direction dependent. Also, during clear days, the diffuse radiation is relatively constant during daytime at about 15% of the peak clear sky radiation until $a > 75^{o}$. For $a > 75^{o}$ the diffuse radiation decreases approximately linearly with a from 75^{o} to 90^{o}. During cloudy conditions, the diffuse radiation is more complicated and measurements are recommended.

Radiation on a horizontal surface

When the direct solar energy is received by a horizontal surface, the horizontal area (A_h) is equal to the object area (A), so the equation simplifies to

$$Q_s = \cos(\alpha)Q$$

The total short wave (solar) radiation received is the sum of the direct (Q_s) and diffuse (q) radiation. However, some of the radiation is reflected, so the net solar radiation (R_{ns}) received is

$R_{ns} = (Q_s + q)(1 - a)$

Where 0 £ a £ 1.0 is the albedo (reflection)

NET RADIATION

Net radiation is the net amount of energy from both short and long wave radiation that is absorbed by a surface.

$$R_n = (Q_s + q)(1 - a) + L_d + L_u$$

where $Q_s = \left(\frac{A_p}{A}\right)Q$ is the direct beam and q is the diffuse solar radiation received by an object and L_d and L_u are long wave downward and upward radiation, respectively. Following our same convention where radiation is positive when added to the surface and negative away from the surface, L_d is positive and L_u is negative.

$$L_d = \varepsilon_a \sigma T_a^4$$

$$L_d = \varepsilon \sigma T_o^4$$

where e_a is the emissivity of the sky, e is the emissivity of the surface, T_a is the effective atmosphere temperature, and T_o is the surface temperature. Because the surface and sky temperature is unknown, the long wave components are estimated from weather station (screen) absolute temperature (T) in Kelvin

$L_u = -esT^4$

$L_d = (1 - c)\, e_a sT^4 + c\,(sT^4 - 9.0)$

where c is the fraction cloud cover. The –9.0 W m^{-2} is included in the right-hand expression to account for the difference in cloud base and weather station temperature. It is assumed that the colder cloud base will emit radiation at about 9.0 W m^{-2} less than if it were at the weather station temperature. In reality, this is an empirical coefficient and it varies depending on the actual cloud base temperature. The cloud cover (c) is typically estimated as a function of the ratio of actual (R_s) to maximum possible solar radiation (R_{so}). However, the relationship is poor because the ratio does not account for cloud distribution and large changes in c correspond to small changes in R_s/R_{so} at near clear sky conditions.

For a grass surface,

e » 0.98

and for clear sky,

$$\varepsilon_a \approx 0.53 + 0.006\sqrt{e}$$

where e is the vapor pressure in Pascals (Pa) measured at screen height

$$e = 610.8 \exp\left(\frac{17.27 T_d}{T_d + 237.3}\right)$$

Here, T_d is the dew point temperature (^oC) at screen height. Note that e_a is not the actual emissivity of the clear sky, but the apparent emissivity resulting from the assumption that $T_a = T$.

REVIEW OF RADIATION TERMS

A = surface area of object

Q_c = energy flux density on a surface normal to the rays w/o an atmosphere

Q = energy flux density on a surface normal to the rays with an atmosphere

A_p = area of object that intercepts radiation (normal to the rays)

A_p/A = interception factor

A_h = area of object shadow on a horizontal surface

A_h/A = shape factor

$R_{ab} = Q\ A_p$ = energy flux or flow to the object

$Q_s = R_{ab}/A = Q\ (A_p/A)$ energy flux to an object divided by the surface area of the object

a = zenith angle is the angle from a line that is perpendicular to a flat (horizontal) surface

For flat (horizontal) surfaces, $A_p/A = \cos(a)$