UC Davis Biometeorology Group

Psychrometer Relationships

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Introduction

These teaching notes discuss psychrometric relationships that are important for understanding humidity and it's measurement. The first section discusses concepts related to measuring dry-bulb temperature. Then concepts pertaining to thermodynamic wet-bulb temperature are presented. Finally, some ideas on measuring the wet bulb temperature are discussed.

Dry-bulb Temperature

The heat transfer to and from a dry-bulb thermometer depends on the long-wave radiation balance and sensible heat transfer. Assuming an emissivity *e* = 1.0, the net long-wave radiation can be expressed in terms of a resistance to radiation transfer (r_R) in s m^{-1}.

$$R_n = \sigma\left(T_s^4 - T_t^4\right) = \rho C_p \frac{\left(T_s - T_t\right)}{r_R} \qquad (1)$$

T_s is the shield temperature (K) and T_t is the thermometer temperature (K). *r* is the air density (kg m^{-3}) and C_p is the specific heat at constant pressure (kJ kg^{-1} ^{o}C^{-1}). The transfer of sensible heat by convection to and from the dry-bulb thermometer inside the shield is given by:

$$H = \rho C_p \left(T_t - T\right) \ r_H^{-1} \qquad (2)$$

H is the sensible heat to and from the thermometer and T is the air temperature within the shield. When in equilibrium, $R_n = H$, so

$$\rho C_p \left(T_s - T_t\right) r_R^{-1} = \rho C_p \left(T_t - T\right) r_H^{-1} \qquad (3)$$

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$$T_t = \frac{r_H T_s + r_R T}{r_R + r_H} \qquad (4)$$

The goal is to have T_t be very close to T. This is accomplished by making:

(1) $r_H <<< r_R$ by using a very small thermometer or by ventilating more

(2) T_s very close to T by painting the shield white, by insulating between outer and inner surfaces, or by increasing ventilation on both sides of the screen.

Thermodynamic Wet-bulb Temperature

In an adiabatic system, the sum of latent and sensible heat remains constant. The initial, energy state is specified by the temperature (T), vapor pressure (e), and total pressure (p). If liquid water is present and e is smaller than $e_s(T)$, then water will evaporate and both e and p will increase. Assuming no exchange of heat between the system and the outside environment, an increase of latent heat in the system, represented by an increase in e, must be balanced by a decrease in sensible heat, represented by a decrease in T. This process will continue until the air becomes saturated at T' (the thermodynamic wet-bulb temperature). The corresponding saturation vapor pressure is $e_s(T')$. The initial water vapor density, defined by T and e, is

$$\varepsilon\left(\frac{e}{p}\right)\rho = \frac{M_v}{M_d}\left(\frac{e}{p}\right)\rho \qquad \text{kg m}^{-3} \qquad (5)$$

where, M_v and M_d are the molecular weights (kg mol^{-1}) of dry air and water vapor, respectively, e is the vapor pressure, p is the barometric pressure, and r is the dry air density. When the vapor pressure rises from e to $e_S(T')$, the change in latent heat per unit volume is

$$\lambda\varepsilon\left(\frac{e_s(T')-e}{p}\right)\rho \qquad \text{kJ m}^{-3} \qquad (6)$$

where *l* is the latent heat of vaporization (kJ kg^{-1}). The corresponding sensible heat used for vaporizing the water is

$$\rho C_p(T-T') \qquad \text{kJ m}^{-3} \qquad (7)$$

Equating the two expressions and solving for e, we get

$$e = e_s(T') - \frac{C_p p}{\lambda\varepsilon}(T-T') \qquad \text{kPa} \qquad (8)$$

where $\gamma = \frac{C_p p}{\lambda\varepsilon}$ is the psychrometric constant. Actually, *g* is not a constant but a function of barometric pressure (p), which depends on elevation and passing weather systems and on the latent heat of vaporization (*l*), which is a weak function of temperature. If p = 101.3 kPa, then *g* = 0.066 at 0^oC and *g* = 0.067 at 20^oC. A good formula to estimate p is

$$p = 101.3\left(\frac{293-0.0065E}{293}\right)^{5.26} \qquad \text{kPa} \qquad (9)$$

To estimate *l*, use

$$\lambda = 2.501 - 0.002361T \qquad \text{J kg}^{-1} \qquad (10)$$

Note that vapor pressure can also be determined from $e_S(T)$ rather than $e_S(T')$. Using the slope of the saturation vapor pressure curve at the mean of the air and wet-bulb temperatures (*D*¢), small changes in saturation vapor pressure are expressed as:

$$e_s(T') = [e_s(T) - \Delta'(T-T')] \qquad \text{kPa} \qquad (11)$$

Substituting into the psychrometric equation (Eq. 8), we get

$$e = e_s(T) - (\Delta' + \gamma)(T-T') \qquad \text{kPa} \qquad (12)$$

Substituting the slope of the saturation vapor pressure curve at air temperature (*D*) for *D*¢, and solving for *T'* provides an equation to estimate the wet-bulb temperature.

$$T' = T - \frac{e_s(T) - e}{\Delta + \gamma} \qquad {}^{o}\mathrm{C} \qquad (13)$$

The error in using *D* rather than *D*¢ is small when *T* – *T'* is small, but the error increases with greater wet-bulb depression.

Measured Wet-bulb Temperature

The measured wet-bulb temperature (T_w) is an estimate of the thermodynamic wet-bulb temperature (*T'*). For a wet-bulb thermometer with temperature (T_w) when exposed to air at temperature (*T*) and surrounded by a radiation shield at air temperature, the rate of sensible heat gain (*H*) and net radiation (R_n) can be expressed as

$$R_n + H = \frac{\rho C_p (T - T_w)}{r_{HR}} \qquad \mathrm{kJ\ m^{-2}s^{-1}} \qquad (14)$$

where r_{HR} is the parallel resistance to convective and radiation heat transfer. The rate of latent heat loss from the wet-bulb is

$$\lambda E = \frac{\lambda \varepsilon \rho}{p}\left(\frac{e_s(T_w) - e}{r_v}\right) \qquad \mathrm{kJ\ m^{-2}s^{-1}} \qquad (15)$$

Recall that the psychrometric constant is

$$\gamma = \frac{C_p p}{\lambda \varepsilon} \quad \text{so} \quad \frac{1}{\gamma} = \frac{\lambda \varepsilon}{C_p p} \quad \text{and} \quad \frac{C_p}{\gamma} = \frac{\lambda \varepsilon}{p} \qquad (16)$$

By substitution, we get

$$\lambda E = \frac{\rho C_p (e_s(T_w) - e)}{\gamma r_v} \qquad (17)$$

In equilibrium, *lE* = R_n + *H*, so

$$\frac{\rho C_p (e_s(T_w) - e)}{\gamma r_v} = \frac{\rho C_p (T - T_w)}{r_{HR}} \qquad (18)$$

then

$$\frac{\rho C_p (e_s(T_w) - e)}{\gamma r_v} = \frac{\rho C_p (T - T_w)}{r_{HR}} \qquad (19)$$

and

$$e = e_s(T_w) - \gamma\left(\frac{r_v}{r_{HR}}\right)(T - T_w) \qquad (20)$$

Thus, the wet-bulb temperature equals the thermodynamic wet-bulb temperature only when $r_v = r_{HR}$. The actual equation to estimate e is

$$e = e_s(T_w) - \gamma^*(T - T_w) \qquad (21)$$

where $\gamma^* = \gamma\left(\frac{r_v}{r_{HR}}\right)$. Based on the concepts of forced convection, the resistance to vapor transfer equals 93% of the resistance to convective heat transfer (r_v = 0.93 r_H) for an aspirated wet-bulb thermometer at 20°C. When $g = g^*$, then the resistance to vapor transfer equals the parallel resistance to convective and radiative heat transfer. Therefore,

$$\frac{1}{r_v} = \frac{1}{r_{HR}} = \frac{1}{r_H} + \frac{1}{r_R} = \frac{1}{0.93 r_H} \qquad (22)$$

Rearranging terms, we get

$$\frac{1}{r_R} = \frac{1 - 0.93}{0.93 r_H} \qquad (23)$$

and

$$r_H = \frac{0.07}{0.93} r_R = 0.075 r_R \qquad (24)$$

For example, if r_R=210 s m^{-1}, then r_H=17 s m^{-1}. Then the measured wet-bulb temperature is bigger than the thermodynamic wet-bulb temperature if r_H>17 s m^{-1}. The measured wet-bulb is smaller when r_H <17 s m^{-1}. Therefore, if a thermometer with r_R=210 s m^{-1} is ventilated so that r_H»17 s m^{-1}, the measured will approximately equal the thermodynamic wet-bulb temperature. The radiation resistance is mainly affected by the thermometer size and shielding. Resistance to sensible heat transfer is mainly affected by ventilation.