Penman-Monteith daily (24-hour) and Hargreaves-Samani Equations for Estimating Reference Evapotranspiration from Monthly Data

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Overview

The following text is a description of the steps needed to estimate reference evapotranspiration (ET_{ref}) for a 0.12 m tall reference surface (ET_{os}) and for a 0.50 m tall reference surface (ET_{rs}) using monthly weather data as adopted by the Environmental Water Resources Institute - American Society of Civil Engineers (ASCE-EWRI, 2004). Note that the steps are in the same sequence as one would use when write computer code. The symbols were shortened to ET_o and ET_r in this documentation. The Hargreaves-Samani (1982); Hargreaves-Samani (1985) equation for estimating ET_o are also presented and the symbol used is ET_h

STEP 1: Extraterrestrial radiation (R_a) is calculated for the midday of each month using the following equations from Duffie and Beckman (1980).

G_{SC} = solar constant in MJ m^{-2} min^{-1}

$G_{SC} = 0.082$

σ = Steffan-Boltzman constant in MJ m^{-2} d^{-1} K^{-4}

$\sigma = 4.90 \times 10^{-9}$

ϕ = latitude in radians converted from latitude (L) in degrees

$$\phi = \frac{\pi L}{180}$$

D_i = number of days per month for i = 1 to 12

M_i = midday of each month i = 1 to 12

$M_1 = 15.5$

$M_i = M_{i-1} + D_i$ for months i=2 to 12

d_r = correction for eccentricity of Earth's orbit around the sun

$$d_r = 1 + 0.033\cos\left(\frac{2\pi}{365}M_i\right) \tag{1}$$

δ = declination of the sun above the celestial equator in radians

$$\delta = 0.409\, sin\left(\frac{2\pi}{365} M_i - 1.39\right) \tag{2}$$

ω_s = sunrise hour angle in radians

$$\omega_s = cos^{-1}\left[-\, tan\,\phi\, tan\,\delta\right] \tag{3}$$

R_a = extraterrestrial radiation (MJ m^{-2} d^{-1})

$$R_a = \left(\frac{24 \cdot 60}{\pi}\right) G_{SC}\, d_r \left[\omega_s \sin\delta \sin\phi + \cos\phi \cos\delta \sin\omega_s\right] \tag{4}$$

STEP 2: Calculate the monthly mean net radiation (R_n) expected over grass in MJ m^{-2} d^{-1} using equations from Allen et al. (1994).

R_{so} = clear sky total global solar radiation at the Earth's surface in MJ m^{-2} d^{-1}

$$R_{so} = R_a\left(0.75 + 2.00 \times 10^{-5} E_L\right) \tag{5}$$

where E_L is the elevation in meters.

R_{ns} = net solar radiation over grass as a function of measured solar radiation (R_s) in MJ m^{-2} d^{-1}

$$R_{ns} = (1 - 0.23) R_s \tag{6}$$

f = a cloudiness function of R_s and R_{so}

$$f = 1.35 \frac{R_s}{R_{so}} - 0.35 \tag{7}$$

$e_s(T_x)$ = saturation vapor pressure (kPa) at the maximum daily air temperature (T_x) in ^{o}C

$$e_s(T_x) = 0.6108 \exp\left[\frac{17.27 T_x}{T_x + 237.3}\right] \tag{8}$$

$e_s(T_n)$ = saturation vapor pressure (kPa) at the minimum daily air temperature (T_n) in ^{o}C

$$e_s(T_n) = 0.6108 \exp\left[\frac{17.27 T_n}{T_n + 237.3}\right] \tag{9}$$

e_a = actual vapor pressure or saturation vapor pressure (kPa) at the mean dew point temperature from daily maximum (RH_x) and minimum (RH_n) relative humidity (%)

$$e_a = \frac{\left(\dfrac{RH_x + RH_n}{2}\right)}{\left(\dfrac{50}{e_s(T_x)} + \dfrac{50}{e_s(T_n)}\right)} \tag{10}$$

e_a = actual vapor pressure or saturation vapor pressure (kPa) at the mean dew point temperature (T_d) in ^{o}C

$$e_a = e_s(T_d) = 0.6108 \exp\left[\frac{17.27 T_d}{T_d + 237.3}\right] \tag{11}$$

ε' = apparent 'net' clear sky emissivity

$$\varepsilon' = 0.34 - 0.14\sqrt{e_a} \tag{12}$$

Note that $\varepsilon' = \varepsilon_{vs} - \varepsilon_a$, where ε_{vs} is the emissivity of the grass and ε_a is the emissivity from the atmosphere. It is called 'apparent' because the temperature from a standard shelter rather than the surface temperature and atmosphere temperature are used to calculate the 'net' long–wave radiation balance. Equation 11 is called the 'Brunt form' equation for net emittance because the form of the equation is similar to Brunt's equation for apparent long-wave emissivity from a clear sky.

R_{nl} = net long wave radiation in MJ m^{-2} d^{-1}

$$R_{nl} = -f\,\varepsilon'\,\sigma\left[\frac{(T_x + 273.15)^4 + (T_n + 273.15)^4}{2}\right] \quad (13)$$

R_n = net radiation over grass in MJ m^{-2} d^{-1}

$$R_n = R_{ns} + R_{nl} \quad (14)$$

STEP 3: Calculate variables needed for the Penman-Monteith equation (ASCE-EWRI, 2004) and the Hargreaves and Samani (1982); Hargreaves and Samani (1985) equation for short canopy reference ET.

β = barometric pressure in kPa as a function of elevation (E_L) in meters

$$\beta = 101.3\left(\frac{293 - 0.0065E_L}{293}\right)^{5.26} \quad (15)$$

λ = latent heat of vaporization in (MJ kg^{-1})

$$\lambda = 2.45 \quad (16)$$

γ = psychrometric constant in kPa $^oC^{-1}$

$$\gamma = 0.00163\frac{\beta}{\lambda} \quad (17)$$

T_m = mean daily temperature in ^oC

$$T_m = \frac{T_x + T_n}{2} \quad (18)$$

G = soil heat flux density in MJ m^{-2} d^{-1}

$$G = 0.07\left(T_{m(i+1)} - T_{m(i-1)}\right) \quad (19)$$

where $T_{m(i-1)}$ and $T_{m(i+1)}$ are the mean daily temperatures for the previous and the following months, respectively.

e^o = saturation vapor pressure at T_m

$$e^o = 0.6108\exp\left(\frac{17.27T_m}{T_m + 237.3}\right) \quad (20)$$

Δ = slope of the saturation vapor pressure curve (kPa $^oC^{-1}$) at mean air temperature (T_m)

$$\Delta = \frac{4099e^o}{(T_m + 237.3)^2} \quad (21)$$

e_s = mean daily saturation vapor pressure (kPa)

$$e_s = \frac{e_s(T_X) + e_s(T_N)}{2} \quad (22)$$

STEP 4: Calculate ET_h using the Hargreaves-Samani (1982); Hargreaves-Samani (1982) equation.

Hargreaves-Samani equation for ET of a short, 0.12 m tall reference surface

$$ET_h = 0.408\left(0.0023R_a[T_m + 17.8]\sqrt{T_x - T_n}\right) \quad (23)$$

where the 0.408 =1/λ factor converts from MJ $m^{-2}d^{-1}$ to mm d^{-1}.

STEP 5: Calculate ET_o using the ASCE-EWRI (2004) standardized equation for short canopy reference *ET*.

R_o = radiation term of the Penman-Monteith equation for short canopy reference *ET* with U_2 the wind speed at 2 m height

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.34U_2)} \tag{24}$$

where 0.408=1/2.45 converts the units from MJ m^{-2} d^{-1} to mm d^{-1}.

A_o = aerodynamic term of the Penman-Monteith equation for short canopy reference *ET* with u_2 the wind speed at 2 m height

$$A_o = \frac{\left(\frac{900\gamma}{T_M + 273}\right)U_2(e_s - e_a)}{\Delta + \gamma(1 + 0.34U_2)} \tag{25}$$

Standardized Reference Evapotranspiration for a short, 0.12 m reference surface in mm d^{-1}.

$$ET_o = R_o + A_o \tag{26}$$

STEP 6: Calculate ET_r using the ASCE-EWRI (2004) standardized equation for tall canopy reference *ET*.

R_r = radiation term of the Penman-Monteith equation for tall canopy reference *ET* with U_2 the wind speed at 2 m height

$$R_r = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.38U_2)} \tag{27}$$

A_r = aerodynamic term of the Penman-Monteith equation for tall canopy reference *ET* with u_2 the wind speed at 2 m height

$$A_r = \frac{\left(\frac{1600\gamma}{T_M + 273}\right)U_2(e_s - e_a)}{\Delta + \gamma(1 + 0.38U_2)} \tag{28}$$

Standardized Reference Evapotranspiration for a tall, 0.5 m reference surface in mm d^{-1}.

$$ET_r = R_r + A_r \tag{29}$$

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