Penman-Monteith (hourly) Reference Evapotranspiration Equations for Estimating ET_{os} and ET_{rs} with Hourly Weather Data

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Overview

The following text is a description of the steps needed to estimate reference evapotranspiration (ET_{ref}) for a 0.12 m tall reference surface (ET_{os}) and for a 0.50 m tall reference surface (ET_{rs}) using hourly weather data as adopted by the Environmental Water Resources Institute - American Society of Civil Engineers (ASCE-EWRI, 2004). Note that the steps are in the same sequence as one would use when write computer code. The steps to calculate the Penman equation estimate of ET_p for a short canopy with no canopy resistance is also provided.

Data Requirements

Site characteristics including the latitude (+ for north and – for south), longitude (+ for west and – for east) and elevation (m) above sea level must be input. The required weather data includes hourly solar radiation (MJ $m^{-2}h^{-1}$), mean air temperature (^oC), mean wind speed (m s^{-1}) and mean dew point temperature (^oC). The air and dew point temperatures should be measured at between 1.5 and 2.0 m height and the wind speed should be measured at 2.0 m height. For wind speeds measured at some height other than 2.0 m, the wind speed at 2 m height (u_2) can be estimated as:

$$u_2 = u_z\left(\frac{4.87}{ln(67.8z_w - 5.42)}\right)$$

where u_z = wind speed (m s^{-1}) at height z_w (m) above the ground.

STEP 1: Extraterrestrial radiation (R_a) is calculated for each hour using the following equations from Duffie and Beckman (1980).

G_{SC} = solar constant in MJ m^{-2} min^{-1}

$G_{SC} = 0.082$

σ = Steffan-Boltzman constant in MJ m^{-2} h^{-1} K^{-4}

$\sigma = 2.04 \times 10^{-10}$

ϕ = Latitude in radians converted from latitude (*L*) in degrees

$$\phi = \frac{\pi L}{180}$$

J = day of the year (1-366)

d_r = correction for eccentricity of Earth's orbit around the sun

$$d_r = 1 + 0.033 \cos\left(\frac{2\pi}{365} J\right) \quad (1)$$

δ = Declination of the sun above the celestial equator in radians

$$\delta = 0.409 \sin\left(\frac{2\pi}{365} J - 1.39\right) \quad (2)$$

L_m = station longitude in degrees

L_z = longitude of the local time meridian

L_z = 120^{o} for Pacific Standard Time

S_c = solar time correction for wobble in Earth's rotation

$$b = \frac{2\pi(J - 81)}{364} \quad (3)$$

$$S_c = 0.1645 \sin(2b) - .1255 \cos(b) - 0.025 \sin(b) \quad (4)$$

t = local standard time (h)

ω = hour angle in radians

$$\omega = \frac{\pi}{12}\left[(t - 0.5) + \frac{L_z - L_m}{15} - 12 + S_c\right] \quad (5)$$

ω_1 = hour angle ½ hour before ω in radians

$$\omega_1 = \omega - \left(\frac{1}{2}\right)\left(\frac{\pi}{12}\right) \quad (6)$$

ω_2 = hour angle ½ hour after ω in radians

$$\omega_2 = \omega + \left(\frac{1}{2}\right)\left(\frac{\pi}{12}\right) \qquad (7)$$

θ = solar altitude angle in radians

$$\sin\theta = (\omega_2 - \omega_1)\sin\phi\sin\delta + \cos\phi\cos\delta(\sin\omega_2 - \sin\omega_1) \qquad (8)$$

R_a = extraterrestrial radiation (MJ m^{-2} h^{-1})

$$R_a = \frac{12}{\pi}(60G_{SC})d_r \sin\theta \qquad (9)$$

β = solar altitude in degrees

$$\beta = \frac{180}{\pi}\sin^{-1}[\sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega] \qquad (10)$$

STEP 2: Calculate the hourly net radiation (R_n) expected over grass in MJ m^{-2} h^{-1} using equations from Allen et al. (1994).

R_{so} = clear sky total global solar radiation at the Earth's surface in MJ m^{-2} h^{-1}

$$R_{so} = R_a\left(0.75 + 2.0\times10^{-5}E_l\right) \qquad (11)$$

where E_l = elevation above mean sea level (m)

e_s = saturation vapor pressure (kPa) at the mean hourly air temperature (T) in ^{o}C

$$e_s = 0.6108\,exp\left[\frac{17.27T}{T + 237.3}\right] \qquad (12)$$

e_a = actual vapor pressure or saturation vapor pressure (kPa) at the mean dew point temperature

$$e_a = 0.6108\,exp\left[\frac{17.27T_d}{T_d + 237.3}\right] \qquad (13)$$

ε' = apparent 'net' clear sky emissivity

$$\varepsilon' = 0.34 - 0.14\sqrt{e_a} \qquad (14)$$

Note that $\varepsilon' = \varepsilon_{vs} - \varepsilon_a$, where ε_{vs} is the emissivity of the grass and ε_a is the emissivity from the atmosphere. It is called 'apparent' because the temperature from a standard shelter rather than the surface temperature and atmosphere temperature are used to calculate the 'net' long–wave radiation balance. Equation 11 is called the 'Brunt form' equation for net emittance because the form of the equation is similar to Brunt's equation for apparent long-wave emissivity from a clear sky.

***f* = a cloudiness function of R_S and R_{SO}**

$$f = 1.35 \frac{R_S}{R_{SO}} - 0.35 \tag{15}$$

with the restriction that $0.3 < R_s / R_{so} \le 1.0$ and $R_s/R_{so} = 0$ whenever $\beta < 17.2^o$ (=0.300 radians) above the horizon. When using a spreadsheet program, put the value $f = 0.6$ in the cell before the first data cell in the column for f. For each sequential hour interval, whenever $\beta < 17.2^o$, let the value for f equal the previous f value. When the corresponding $\beta \ge 17.2^o$, use the R_s / R_{so} and Equation 15 to calculate the f values. The values for f will fall between 0.05 and 1.00. If this procedure is followed, the nighttime values for f will equal the f value at the end of the previous daylight period until the next daylight period. The nighttime f values are used to estimate the effect of cloud cover on R_n during the night. This method is used in the PMhr.xls program.

R_{ns} = net short wave radiation as a function of measured solar radiation (R_s) in MJ m^{-2} h^{-1}

$$R_{ns} = (1 - 0.23) R_s \tag{16}$$

To convert R_s from W m^{-2} to MJ m^{-2} h^{-1}, multiply by 0.0036.

R_{nl} = net long wave radiation in MJ m^{-2} h^{-1}

$$R_{nl} = -f \varepsilon' \sigma (T + 273.15)^4 \tag{17}$$

R_n = net radiation over grass in MJ m^{-2} h^{-1}

$$R_n = R_{ns} + R_{nl} \tag{18}$$

STEP 3: Calculate ET_o using the Penman-Monteith equation as presented by Allen et al. (1994)

B_p = barometric pressure in kPa as a function of elevation (E_l) in meters

$$B_p = 101.3 \left(\frac{293 - 0.0065 E_l}{293} \right)^{5.26} \tag{19}$$

λ = latent heat of vaporization in (MJ kg^{-1})

$$\lambda = 2.45 \tag{20}$$

γ = psychrometric constant in kPa $^{o}C^{-1}$

$$\gamma = 0.00163 \frac{B_p}{\lambda} \tag{21}$$

r_a = aerodynamic resistance in s m^{-1} is estimated for a 0.12 m tall crop as a function of wind speed (u_2) in m s^{-1} as:

$$r_a = \frac{208}{u_2} \tag{22}$$

Modified psychrometric constant (γ*)

For the short 0.12 m tall canopy during daylight (when $R_n > 0$), a canopy resistance of r_s = 50 s m^{-1} and an aerodynamic resistance of $r_a = 208/u_2$ are used to calculate modified psychrometric constant as:

$$\gamma^* = \gamma\left(1 + \frac{r_s}{r_a}\right) \approx \gamma(1 + 0.24u_2) \tag{23}$$

During the night (when $R_n \leq 0$), a canopy resistance of r_s = 200 s m^{-1} and an aerodynamic resistance of $r_a = 208/u_2$ γ^* are used to calculate the modified psychrometric constant as:

$$\gamma^* = \gamma\left(1 + \frac{r_s}{r_a}\right) \approx \gamma(1 + 0.96u_2) \tag{24}$$

For wind speeds less than 0.5 m s^{-1}, the wind speed is set equal to 0.5 m s^{-1} for both Eqs. 23 and 24. For the 0.50 m tall canopy during daylight (when $R_n > 0$), a canopy resistance of r_s = 30 s m^{-1} and an aerodynamic resistance of $r_a = 118/u_2$ s m^{-1} are used to calculate the modified psychrometric constant as:

$$\gamma^* = \gamma\left(1 + \frac{r_s}{r_a}\right) \approx \gamma(1 + 0.25u_2) \tag{25}$$

During the night (when $R_n \leq 0$), a canopy resistance of r_s = 200 s m^{-1} and an aerodynamic resistance of $r_a = 118/u_2$ s m^{-1} are used to calculate the modified psychrometric constant as:

$$\gamma^* = \gamma\left(1 + \frac{r_s}{r_a}\right) \approx \gamma(1 + 1.7u_2) \tag{26}$$

For wind speeds less than 0.5 m s^{-1}, the wind speed is set equal to 0.5 m s^{-1} for both Eqs. 25 and 26.

Δ = slope of the saturation vapor pressure curve (kPa $^{o}C^{-1}$) at mean air temperature (T)

$$\Delta = \frac{4099 e_s}{(T + 237.3)^2} \tag{27}$$

G = soil heat flux density (MJ m^{-2} h^{-1})

For ET_{os}, let $G = 0.1\ R_n$ when $R_n > 0$ and let $G = 0.5\ R_n$ for $R_n < 0$. For ET_{rs}, let $G = 0.04\ R_n$ when $R_n > 0$ and $G = 0.2\ R_n$ when $R_n \leq 0$.

R is the radiation term of the Penman-Monteith and Penman equations in mm d^{-1} .

When $R_n > 0$, for ET_{os}, the radiation term contribution to ET is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.24U_2)} \qquad (28)$$

And during the night, it is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.96U_2)} \qquad (29)$$

When $R_n > 0$, for ET_{rs}, the radiation term contribution to ET is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.25U_2)} \qquad (30)$$

And during the night, it is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 1.7U_2)} \qquad (31)$$

For the ET_p (Penman equation), the radiation term contribution to ET is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma} \qquad (32)$$

for both day and night calculations.

A = aerodynamic term of the Penman-Monteith equation in mm d^{-1} with u_2 the wind speed at 2 m height

When $R_n > 0$, for ET_{os}, the aerodynamic contribution to ET is calculated as:

$$A_o = \frac{\left(\frac{37\gamma}{T_M + 273}\right)u_2(e_s - e_a)}{\Delta + \gamma(1 + 0.24u_2)} \qquad (33)$$

And during the night, it is calculated as:

$$A_o = \frac{\left(\frac{37\gamma}{T_M + 273}\right) u_2 (e_s - e_a)}{\Delta + \gamma(1 + 0.96 u_2)} \tag{34}$$

When $R_n > 0$, for ET_{rs}, the aerodynamic contribution to ET is calculated as:

$$A_r = \frac{\left(\frac{66\gamma}{T_M + 273}\right) u_2 (e_s - e_a)}{\Delta + \gamma(1 + 0.25 u_2)} \tag{35}$$

And during the night, it is calculated as:

$$A_r = \frac{\left(\frac{66\gamma}{T_M + 273}\right) u_2 (e_s - e_a)}{\Delta + \gamma(1 + 1.7 u_2)} \tag{36}$$

For ET_p, the aerodynamic contribution to ET during daytime and nighttime is calculated as:

$$A_p = \frac{\left(\frac{37\gamma}{T_M + 273}\right) u_2 (e_s - e_a)}{\Delta + \gamma} \tag{37}$$

Reference evapotranspiration

For a short (0.12 m) canopy, the Penman-Monteith reference evapotranspiration is calculated as:

$$ET_{os} = R_o + A_o \tag{38}$$

Similarly, for a tall (0.5 m) canopy, the Penman-Montieth reference evapotranspiration is calculated as:

$$ET_{rs} = R_r + A_r \tag{39}$$

For a short (0.12 m) tall canopy, the Penman evapotranspiration is calculated as:

$$ET_{os} = R_o + A_o \tag{40}$$

In equations 38-40, the units are mm h^{-1}.

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