

Penman-Monteith (hourly) Reference Evapotranspiration Equations for Estimating ET_{os} and ET_{rs} with Hourly Weather Data

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Overview

The following text is a description of the steps needed to estimate reference evapotranspiration (ET_{ref}) for a 0.12 m tall reference surface (ET_{os}) and for a 0.50 m tall reference surface (ET_{rs}) using hourly weather data as adopted by the Environmental Water Resources Institute - American Society of Civil Engineers (ASCE-EWRI, 2004). Note that the steps are in the same sequence as one would use when write computer code. The steps to calculate the Penman equation estimate of ET_p for a short canopy with no canopy resistance is also provided.

Data Requirements

Site characteristics including the latitude (+ for north and – for south), longitude (+ for west and – for east) and elevation (m) above sea level must be input. The required weather data includes hourly solar radiation ($\text{MJ m}^{-2}\text{h}^{-1}$), mean air temperature ($^{\circ}\text{C}$), mean wind speed (m s^{-1}) and mean dew point temperature ($^{\circ}\text{C}$). The air and dew point temperatures should be measured at between 1.5 and 2.0 m height and the wind speed should be measured at 2.0 m height. For wind speeds measured at some height other than 2.0 m, the wind speed at 2 m height (u_2) can be estimated as:

$$u_2 = u_z \left(\frac{4.87}{\ln(67.8z_w - 5.42)} \right)$$

where u_z = wind speed (m s^{-1}) at height z_w (m) above the ground.

STEP 1: Extraterrestrial radiation (R_a) is calculated for each hour using the following equations from Duffie and Beckman (1980).

G_{SC} = solar constant in $\text{MJ m}^{-2} \text{min}^{-1}$

$$G_{SC} = 0.082$$

σ = Steffan-Boltzman constant in MJ m⁻² h⁻¹ K⁻⁴

$$\sigma = 2.04 \times 10^{-10}$$

ϕ = Latitude in radians converted from latitude (L) in degrees

$$\phi = \frac{\pi L}{180}$$

J = day of the year (1-366)

d_r = correction for eccentricity of Earth's orbit around the sun

$$d_r = 1 + 0.033 \cos\left(\frac{2\pi}{365} J\right) \quad (1)$$

δ = Declination of the sun above the celestial equator in radians

$$\delta = 0.409 \sin\left(\frac{2\pi}{365} J - 1.39\right) \quad (2)$$

L_m = station longitude in degrees

L_z = longitude of the local time meridian

$L_z = 120^\circ$ for Pacific Standard Time

S_c = solar time correction for wobble in Earth's rotation

$$b = \frac{2\pi(J - 81)}{364} \quad (3)$$

$$S_c = 0.1645 \sin(2b) - .1255 \cos(b) - 0.025 \sin(b) \quad (4)$$

t = local standard time (h)

ω = hour angle in radians

$$\omega = \frac{\pi}{2} \left[(t - 0.5) + \frac{L_z - L_m}{15} - 12 + S_c \right] \quad (5)$$

ω_1 = hour angle 1/2 hour before ω in radians

$$\omega_1 = \omega - \left(\frac{1}{2}\right) \left(\frac{\pi}{12}\right) \quad (6)$$

ω_2 = hour angle $\frac{1}{2}$ hour after ω in radians

$$\omega_1 = \omega + \left(\frac{1}{2}\right)\left(\frac{\pi}{12}\right) \quad (7)$$

θ = solar altitude angle in radians

$$\sin \theta = (\omega_2 - \omega_1) \sin \phi \sin \delta + \cos \phi \cos \delta (\sin \omega_2 - \sin \omega_1) \quad (8)$$

R_a = extraterrestrial radiation ($\text{MJ m}^{-2} \text{h}^{-1}$)

$$R_a = \frac{12}{\pi} (60G_{SC}) d_r \sin \theta \quad (9)$$

β = solar altitude in degrees

$$\beta = \frac{180}{\pi} \sin^{-1} [\sin \phi \sin \delta + \cos \phi \cos \delta \cos \omega] \quad (10)$$

STEP 2: Calculate the hourly net radiation (R_n) expected over grass in $\text{MJ m}^{-2} \text{h}^{-1}$ using equations from Allen et al. (1994).

R_{so} = clear sky total global solar radiation at the Earth's surface in $\text{MJ m}^{-2} \text{h}^{-1}$

$$R_{so} = R_a (0.75 + 2.0 \times 10^{-5} E_l) \quad (11)$$

where E_l = elevation above mean sea level (m)

e_s = saturation vapor pressure (kPa) at the mean hourly air temperature (T) in $^{\circ}\text{C}$

$$e_s = 0.6108 \exp \left[\frac{17.27T}{T + 237.3} \right] \quad (12)$$

e_a = actual vapor pressure or saturation vapor pressure (kPa) at the mean dew point temperature

$$e_a = 0.6108 \exp \left[\frac{17.27T_d}{T_d + 237.3} \right] \quad (13)$$

ε' = apparent 'net' clear sky emissivity

$$\varepsilon' = 0.34 - 0.14 \sqrt{e_a} \quad (14)$$

Note that $\varepsilon' = \varepsilon_{vs} - \varepsilon_a$, where ε_{vs} is the emissivity of the grass and ε_a is the emissivity from the atmosphere. It is called ‘apparent’ because the temperature from a standard shelter rather than the surface temperature and atmosphere temperature are used to calculate the ‘net’ long-wave radiation balance. Equation 11 is called the ‘Brunt form’ equation for net emittance because the form of the equation is similar to Brunt’s equation for apparent long-wave emissivity from a clear sky.

f = a cloudiness function of R_s and R_{s0}

$$f = 1.35 \frac{R_s}{R_{s0}} - 0.35 \quad (15)$$

with the restriction that $0.35 < f \leq 1.0$.

On the hour when the sun first falls below a selected angle above the horizon, f = the f value of the previous hour. The angle is input by the user with a default value of 17.2° ($=0.300$ radians) above the horizon. In mountainous regions, a higher value for the angle above the horizon can be input to override the default. After the sun angle falls below the selected value, f remains the same all night until the f calculated with equation 15 exceeds the nighttime f value the next morning. Note that this is the method used in the PMhr.xls program as well as the HRPM.exe program.

R_{ns} = net short wave radiation as a function of measured solar radiation (R_s) in $\text{MJ m}^{-2} \text{h}^{-1}$

$$R_{ns} = (1 - 0.23)R_s \quad (16)$$

To convert R_s from W m^{-2} to $\text{MJ m}^{-2} \text{h}^{-1}$, multiply by 0.0036.

R_{nl} = net long wave radiation in $\text{MJ m}^{-2} \text{h}^{-1}$

$$R_{nl} = -f\varepsilon'\sigma(T + 273.15)^4 \quad (17)$$

R_n = net radiation over grass in $\text{MJ m}^{-2} \text{h}^{-1}$

$$R_n = R_{ns} + R_{nl} \quad (18)$$

STEP 3: Calculate ET_o using the Penman-Monteith equation as presented by Allen et al. (1994)

B_p = barometric pressure in kPa as a function of elevation (E_l) in meters

$$B_p = 101.3 \left(\frac{293 - 0.0065E_l}{293} \right)^{5.26} \quad (19)$$

λ = latent heat of vaporization in (MJ kg^{-1})

$$\lambda = 2.45 \quad (20)$$

γ = psychrometric constant in kPa °C⁻¹

$$\gamma = 0.00163 \frac{B_p}{\lambda} \quad (21)$$

r_a = aerodynamic resistance in s m⁻¹ is estimated for a 0.12 m tall crop as a function of wind speed (u_2) in m s⁻¹ as:

$$r_a = \frac{208}{u_2} \quad (22)$$

Modified psychrometric constant (γ^*)

For the short 0.12 m tall canopy during daylight (when $R_n > 0$), a canopy resistance of $r_s = 50$ s m⁻¹ and an aerodynamic resistance of $r_a = 208/u_2$ are used to calculate modified psychrometric constant as:

$$\gamma^* = \gamma \left(1 + \frac{r_s}{r_a} \right) \approx \gamma(1 + 0.24u_2) \quad (23)$$

During the night (when $R_n \leq 0$), a canopy resistance of $r_s = 200$ s m⁻¹ and an aerodynamic resistance of $r_a = 208/u_2$ γ^* are used to calculate the modified psychrometric constant as:

$$\gamma^* = \gamma \left(1 + \frac{r_s}{r_a} \right) \approx \gamma(1 + 0.96u_2) \quad (24)$$

For wind speeds less than 0.5 m s⁻¹, the wind speed is set equal to 0.5 m s⁻¹ for both Eqs. 23 and 24. For the 0.50 m tall canopy during daylight (when $R_n > 0$), a canopy resistance of $r_s = 30$ s m⁻¹ and an aerodynamic resistance of $r_a = 118/u_2$ s m⁻¹ are used to calculate the modified psychrometric constant as:

$$\gamma^* = \gamma \left(1 + \frac{r_s}{r_a} \right) \approx \gamma(1 + 0.25u_2) \quad (25)$$

During the night (when $R_n \leq 0$), a canopy resistance of $r_s = 200$ s m⁻¹ and an aerodynamic resistance of $r_a = 118/u_2$ s m⁻¹ are used to calculate the modified psychrometric constant as:

$$\gamma^* = \gamma \left(1 + \frac{r_s}{r_a} \right) \approx \gamma(1 + 1.7u_2) \quad (26)$$

For wind speeds less than 0.5 m s⁻¹, the wind speed is set equal to 0.5 m s⁻¹ for both Eqs. 25 and 26.

Δ = slope of the saturation vapor pressure curve (kPa °C⁻¹) at mean air temperature (T)

$$\Delta = \frac{4099e_s}{(T + 237.3)^2} \quad (27)$$

G = soil heat flux density (MJ m⁻² h⁻¹)

For ET_{os} , let $G = 0.1 R_n$ when $R_n > 0$ and let $G = 0.5 R_n$ for $R_n < 0$. For ET_{rs} , let $G = 0.04 R_n$ when $R_n > 0$ and $G = 0.2 R_n$ when $R_n \leq 0$.

R is the radiation term of the Penman-Monteith and Penman equations in mm d⁻¹.

When $R_n > 0$, for ET_{os} , the radiation term contribution to ET is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.24U_2)} \quad (28)$$

And during the night, it is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.96U_2)} \quad (29)$$

When $R_n > 0$, for ET_{rs} , the radiation term contribution to ET is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.25U_2)} \quad (30)$$

And during the night, it is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 1.7U_2)} \quad (31)$$

For the ET_p (Penman equation), the radiation term contribution to ET is calculated as:

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma} \quad (32)$$

for both day and night calculations.

A = aerodynamic term of the Penman-Monteith equation in mm d⁻¹ with u_2 the wind speed at 2 m height

When $R_n > 0$, for ET_{os} , the aerodynamic contribution to ET is calculated as:

$$A_o = \frac{\left(\frac{37\gamma}{T_M + 273}\right)u_2(e_s - e_a)}{\Delta + \gamma(1 + 0.24u_2)} \quad (33)$$

And during the night, it is calculated as:

$$A_o = \frac{\left(\frac{37\gamma}{T_M + 273}\right)u_2(e_s - e_a)}{\Delta + \gamma(1 + 0.96u_2)} \quad (34)$$

When $R_n > 0$, for ET_{rs} , the aerodynamic contribution to ET is calculated as:

$$A_r = \frac{\left(\frac{66\gamma}{T_M + 273}\right)u_2(e_s - e_a)}{\Delta + \gamma(1 + 0.25u_2)} \quad (35)$$

And during the night, it is calculated as:

$$A_r = \frac{\left(\frac{66\gamma}{T_M + 273}\right)u_2(e_s - e_a)}{\Delta + \gamma(1 + 1.7u_2)} \quad (36)$$

For ET_p , the aerodynamic contribution to ET during daytime and nighttime is calculated as:

$$A_p = \frac{\left(\frac{37\gamma}{T_M + 273}\right)u_2(e_s - e_a)}{\Delta + \gamma} \quad (37)$$

Reference evapotranspiration

For a short (0.12 m) canopy, the Penman-Monteith reference evapotranspiration is calculated as:

$$ET_{os} = R_o + A_o \quad (38)$$

Similarly, for a tall (0.5 m) canopy, the Penman-Monteith reference evapotranspiration is calculated as:

$$ET_{rs} = R_r + A_r \quad (39)$$

For a short (0.12 m) tall canopy, the Penman evapotranspiration is calculated as:

$$ET_{os} = R_o + A_o \quad (40)$$

In equations 38-40, the units are mm h^{-1} .

REFERENCES

Allen, R.G., M.E., Jensen, J.L. Wright, and R.D. Burman. 1989. Operational estimates of evapotranspiration. *Agron. J.* 81:650-662.

- Allen, R.G. and W.O. Pruitt. 1991. FAO-24 Reference evapotranspiration factors. *J. of Irrig. and Drainage Engineering*. 117(5):758-773.
- Allen, R.G., M. Smith, L.S. Pereira, A. Perrier. 1994. An update for the calculation of reference evapotranspiration. *ICID Bulletin 1994 Vol 43 No 2*.
- Allen, R.G., Walter, I.A., Elliott, R.L., Howell, T.A., Itenfisu, D., Jensen, M.E. and Snyder, R.L. 2005. The ASCE Standardized Reference Evapotranspiration Equation. Amer. Soc. of Civil Eng. Reston, Virginia. 192p.
- Doorenbos, J. and W.O. Pruitt. 1977. Crop Water Requirements. FAO Irrigation and Drainage Paper 24, United Nation Food and Agriculture Organization, Rome.
- Duffie, J.A. and W.A. Beckman. 1980. Solar engineering of thermal processes. John Wiley and Sons, New York. pp. 1-109.
- Jensen, M.E., R.D. Burman, and R.G. Allen, Eds. 1990. Evapotranspiration and Irrigation Water Requirements. Amer. Soc. of Civil Eng., New York.
- Smith, M. 1991. Report on the expert consultation on procedures for revision of FAO Guidelines for prediction of crop water requirements. United Nations - Food and Agriculture Organization, Rome, Italy
- Tetens, V.O. 1930. Uber einige meteorologische. Begriffe, *Zeitschrift fur Geophysik*. 6:297-309.
- Walter, I.A., R.G. Allen, R. Elliott, M.E. Jensen, D. Itenfisu, B. Mecham, T.A. Howell, R. Snyder, P. Brown, S. Eching, T. Spofford, M. Hattendorf, R.H. Cuenca, J.L. Wright, D. Martin. 2000. ASCE's Standardized Reference Evapotranspiration Equation. Proc. of the Watershed Management 2000 Conference, June 2000, Ft. Collins, CO, American Society of Civil Engineers, St. Joseph, MI.