Penman-Monteith daily (24-hour) Reference Evapotranspiration Equations for Estimating ET_o, ET_r and HS ET_o with Daily Data

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Overview

The following text is a description of the steps needed to estimate reference evapotranspiration (ET_{ref}) for a 0.12 m tall reference surface (ET_{os}) and for a 0.50 m tall reference surface (ET_{rs}) using daily weather data as adopted by the Environmental Water Resources Institute - American Society of Civil Engineers (ASCE-EWRI, 2004). Note that the steps are in the same sequence as one would use when write computer code. The symbols were shortened to ET_o and ET_r in this documentation. The Hargreaves-Samani (1982); Hargreaves-Samani (1985) equation for estimating ET_o are also presented and the symbol used is ET_h

Data Requirements

Site characteristics including the latitude (+ for north and – for south) and elevation (m) above sea level must be input. The required weather data includes hourly solar radiation (MJ $m^{-2}d^{-1}$), maximum and minimum air temperature (^oC), mean wind speed (m s^{-1}) and mean dew point temperature (^oC). The air and dew point temperatures should be measured at between 1.5 and 2.0 m height and the wind speed should be measured at 2.0 m height. For wind speeds measured at some height other than 2.0 m, the wind speed at 2 m height (u_2) can be estimated as:

$$u_2 = u_z\left(\frac{4.87}{\ln(67.8z_w - 5.42)}\right)$$

where u_z = wind speed (m s^{-1}) at height z_w (m) above the ground.

The following text is a description of the steps needed to estimate reference evapotranspiration for a 0.12 m tall reference surface using daily weather data as adopted by the American Society of Civil Engineers (ASCE-EWRI, 2004). The steps are in the same sequence as one would enter code into a computer program.

STEP 1: Extraterrestrial radiation (R_a) is calculated for each day using the following equations from Duffie and Beckman (1980).

G_{SC} = solar constant in MJ m^{-2} min^{-1}

$$G_{SC} = 0.082$$

σ = Steffan-Boltzman constant in MJ m^{-2} d^{-1} K^{-4}

$$\sigma = 4.90 \times 10^{-9}$$

ϕ = latitude in radians converted from latitude (L) in degrees

$$\phi = \frac{\pi L}{180}$$

d_r = correction for eccentricity of Earth's orbit around the sun on day i of the year

$$d_r = 1 + 0.033 cos\left(\frac{2\pi}{365} i\right) \quad (1)$$

δ = declination of the sun above the celestial equator in radians on day i of the year

$$\delta = 0.409 \, sin\left(\frac{2\pi}{365} i - 1.39\right) \quad (2)$$

ω_s = sunrise hour angle in radians

$$\omega_s = cos^{-1}\left[- tan\,\phi \, tan\,\delta\right] \quad (3)$$

R_a = extraterrestrial radiation (MJ m^{-2} d^{-1})

$$R_a = \left(\frac{24 \cdot 60}{\pi}\right) G_{SC} d_r \left[\omega_s \sin\delta \sin\phi + \cos\phi \cos\delta \sin\omega_s\right] \quad (4)$$

STEP 2: Calculate the daily net radiation (R_n) expected over grass in MJ m^{-2} d^{-1} using equations from Allen et al. (1994).

R_{so} = clear sky total global solar radiation at the Earth's surface in MJ m^{-2} d^{-1}

$$R_{so} = R_a\left(0.75 + 2.0 \times 10^{-5} E_l\right) \quad (5)$$

R_{ns} = net solar radiation over grass as a function of measured solar radiation (R_S) in MJ m^{-2} d^{-1}

$$R_{ns} = (1 - 0.23) R_s \quad (6)$$

f = a cloudiness function of R_s and R_{so}

$$f = 1.35 \frac{R_s}{R_{so}} - 0.35 \quad (7)$$

$e_s(T_x)$ = saturation vapor pressure (kPa) at the maximum daily air temperature (T_X) in ^{o}C

$$e_s(T_x) = 0.6108 \exp\left(\frac{17.27 T_x}{T_x + 237.3}\right) \quad (8)$$

$e_s(T_n)$ = saturation vapor pressure (kPa) at the minimum daily air temperature (T_n) in ^{o}C

$$e_s(T_n) = 0.6108 \exp\left(\frac{17.27 T_n}{T_n + 237.3}\right) \quad (9)$$

e_d = actual vapor pressure or saturation vapor pressure (kPa) at the mean dew point temperature from the daily maximum (T_x) and minimum (T_n) temperature (^{o}C) and maximum (RH_x) and minimum (RH_n) relative humidity (%).

$$e_d = \frac{e_s(T_x)\frac{RH_n}{100} + e_s(T_n)\frac{RH_x}{100}}{2} \quad (10)$$

e_d = actual vapor pressure or saturation vapor pressure (kPa) at the daily mean dew point (T_d) temperature.

$$e_d = e_s(T_d) = 0.6108\exp\left(\frac{17.27T_d}{T_d + 237.3}\right) \quad (11)$$

ε' = apparent 'net' clear sky emissivity

$$\varepsilon' = 0.34 - 0.14\sqrt{e_a} \quad (12)$$

Note that $\varepsilon' = \varepsilon_{vs} - \varepsilon_a$, where ε_{vs} is the emissivity of the grass and ε_a is the emissivity from the atmosphere. It is called 'apparent' because the temperature from a standard shelter rather than the surface temperature and atmosphere temperature are used to calculate the 'net' long–wave radiation balance. Equation 11 is called the 'Brunt form' equation for net emittance because the form of the equation is similar to Brunt's equation for apparent long-wave emissivity from a clear sky.

R_{nl} = net long wave radiation in MJ m^{-2} d^{-1}

$$R_{nl} = -f\,\varepsilon'\,\sigma\left[\frac{(T_x + 273.15)^4 + (T_n + 273.15)^4}{2}\right] \quad (13)$$

R_n = net radiation over grass in MJ m^{-2} d^{-1}

$$R_n = R_{ns} + R_{nl} \quad (14)$$

STEP 3: Calculate variables needed to compute ET_h, ET_o and ET_r.

β = barometric pressure in kPa as a function of elevation (E_l) in meters

$$\beta = 101.3\left(\frac{293 - 0.0065E_l}{293}\right)^{5.26} \quad (15)$$

λ = latent heat of vaporization in (MJ kg^{-1})

$$\lambda = 2.45 \quad (16)$$

γ = psychrometric constant in kPa $^oC^{-1}$

$$\gamma = 0.00163\frac{\beta}{\lambda} \quad (17)$$

T_m = mean daily temperature in ^oC

$$T_m = \frac{T_x + T_n}{2} \quad (18)$$

e^o = saturation vapor pressure at T_m

$$e^o = 0.6108\exp\left(\frac{17.27T_m}{T_m + 237.3}\right) \quad (19)$$

Δ = slope of the saturation vapor pressure curve (kPa $^oC^{-1}$) at mean air temperature (T_m)

$$\Delta = \frac{4099e^o}{(T_m + 237.3)^2} \quad (20)$$

G = soil heat flux density in MJ m^{-2} d^{-1}

$$G \approx 0 \quad (21)$$

e_a = mean of the T_x and T_n saturation vapor pressures (kPa)

$$e_a = \frac{e_s(T_x) + e_s(T_n)}{2} \quad (22)$$

STEP 4: Calculate *ET_h* using the Hargreaves-Samani (1982); Hargreaves-Samani (1982) equation.

Hargreaves-Samani equation for ET of a short, 0.12 m tall reference surface

$$ET_h = 0.408\left(0.0023 R_a [T_m + 17.8]\sqrt{T_x - T_n}\right) \quad (23)$$

where the 0.408 =1/λ factor converts from MJ $m^{-2}d^{-1}$ to mm d^{-1}.

STEP 5: Calculate *ET_o* using the ASCE-EWRI (2004) standardized equation for short canopy reference *ET*.

R_o = radiation term of the Penman-Monteith equation for short canopy reference *ET* with U_2 the wind speed at 2 m height

$$R_o = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.34U_2)} \quad (24)$$

where 0.408=1/2.45 converts the units from MJ m^{-2} d^{-1} to mm d^{-1}.

A_o = aerodynamic term of the Penman-Monteith equation for short canopy reference *ET* with u_2 the wind speed at 2 m height

$$A_o = \frac{\left(\frac{900\gamma}{T_M + 273}\right)U_2(e_a - e_d)}{\Delta + \gamma(1 + 0.34U_2)} \quad (25)$$

Standardized Reference Evapotranspiration for a short, 0.12 m reference surface in mm d^{-1}.

$$ET_o = R_o + A_o \quad (26)$$

STEP 6: Calculate *ET_r* using the ASCE-EWRI (2004) standardized equation for tall canopy reference *ET*.

R_r = radiation term of the Penman-Monteith equation for tall canopy reference *ET* with U_2 the wind speed at 2 m height

$$R_r = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.38U_2)} \quad (27)$$

A_r = aerodynamic term of the Penman-Monteith equation for tall canopy reference *ET* with u_2 the wind speed at 2 m height

$$A_r = \frac{\left(\frac{1600\gamma}{T_M + 273}\right)U_2(e_a - e_d)}{\Delta + \gamma(1 + 0.38U_2)} \quad (28)$$

Standardized Reference Evapotranspiration for a tall, 0.5 m reference surface in mm d^{-1}.

$$ET_r = R_r + A_r \quad (29)$$

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